Analog Electronics
ENEE236

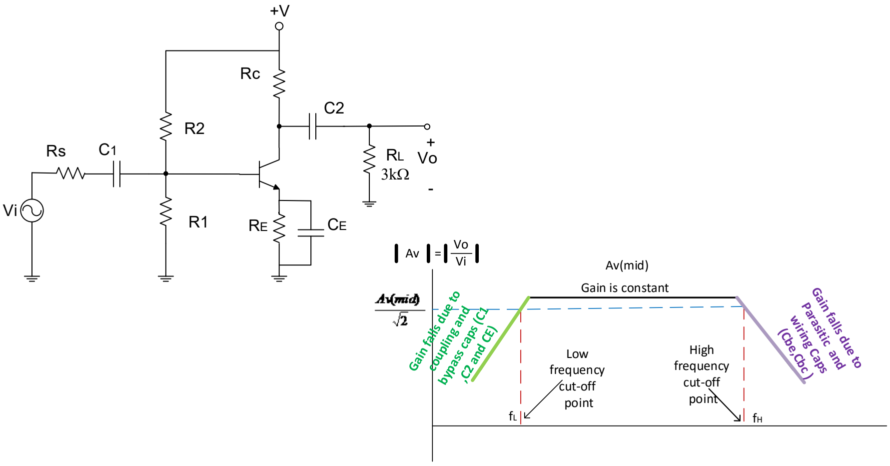
Amplifiers Frequency Response

Instructor
Nasser Ismail

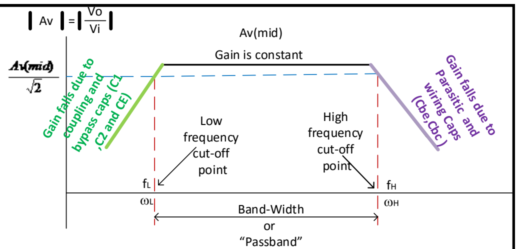
Amplifier Frequency Response

- Audio frequency signals such as speech and music are combination of many different sine waves, occurring simultaneously with different amplitude and frequency in the following range (20Hz-20kHz (audible noise) , other types of signals have their own range.
- In order for the output to be an amplified version of the input, the amplifier must amplify each and every component in the signal by the same amount
- The Bandwidth must cover the entire range of frequency components if considered amplification is to be achieved

Amplifier Frequency Response



Impedance of a cap



- The impedance of a cap is

$$X_c = \frac{1}{2\pi f C}$$

when $f < f_L$ the coupling caps C1 and C2, and the bypass cap CE cannot be considered as short circuit since their impedance is not small enough

when $f > f_H$ the internal caps Cbc and Cbe for a BJT (or Cgs and Cgd), cannot be considered as open circuit since their impedance is not high enough

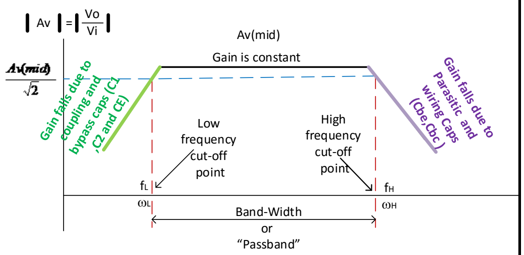
Corner Frequency

we define the corner (break and cut - off) frequency as :

$$|A(j\omega_L)| = \frac{Av(mid)}{\sqrt{2}}$$

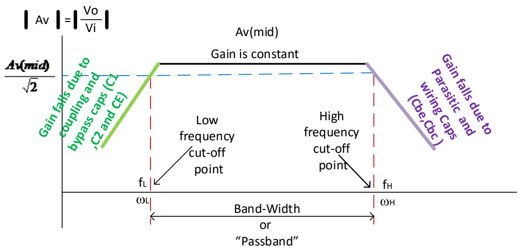
$$|A(j\omega_H)| = \frac{Av(mid)}{\sqrt{2}}$$

$$\omega_H - \omega_L = BW \text{ - Bandwidth}$$



$$\text{Midrange} = \text{midband} \cong 10\omega_L - 0.1\omega_H$$

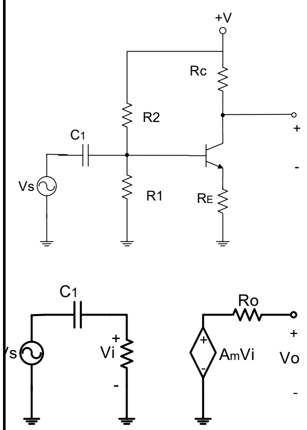
Corner Frequency



$$\text{Midrange} = \text{midband} \cong 10\omega_L - 0.1\omega_H$$

– This is the range for which the capacitors (C1, C2 and CE) are considered short circuit while the parasitic caps are considered open circuit (this is the range we have considered so far in previuos chapters)

Series Capacitance and low frequency response



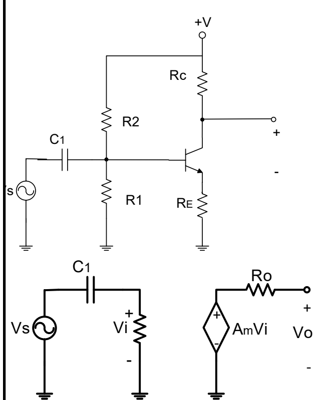
$$Vo = AmVi$$

$$Vi = \frac{Ri}{Ri + \frac{1}{j\omega C1}} Vs \rightarrow Vo = \frac{Am.Ri}{Ri + \frac{1}{j\omega C1}} Vs$$

$$\frac{Vo}{V_s} = \frac{Am.Ri}{Ri + \frac{1}{j\omega C1}} = A(j\omega)$$

$$\rightarrow |A(j\omega)| = \frac{Am.Ri}{\sqrt{(Ri)^2 + \left(\frac{1}{\omega C1}\right)^2}}$$

Series Capacitance and low frequency response



$$|A(j\omega)| = \frac{Am.Ri}{\sqrt{(Ri)^2 + \left(\frac{1}{\omega C1}\right)^2}}$$

$$= \frac{Am}{\sqrt{1 + \left(\frac{\omega_{C1}}{\omega}\right)^2}};$$

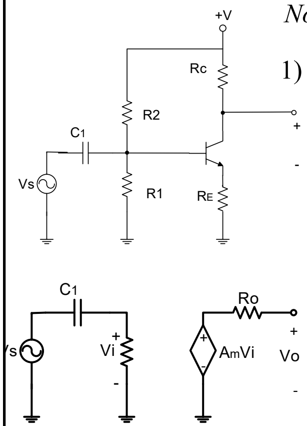
$$where \ \omega_{C1} = \frac{1}{Ri.C1}$$ is the break frequency due to C1

For $Am = 1$

for $\omega = \omega_{C1} \rightarrow 20\log|A(j\omega)| = 20\log Am - 20\log 0.707 = -3dB$

for $\omega = 0.1\omega_{C1} \rightarrow 20\log|A(j\omega)| = -20\log 10 = -20dB \leftarrow$ High pass filter

Series Capacitance and low frequency response



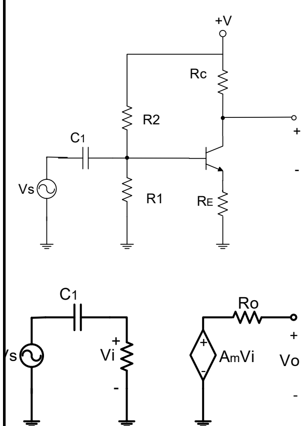
Note:

1) If there is only one cap, we find $\omega_{C1} = \dfrac{1}{Rth1.C1}$ and $\omega_L = \omega_{C1}$

2) If there is two caps C1 and C2 with ω_{C1} and ω_{C2} , then

$$A(j\omega) = Am\left(\frac{1}{1+\left(\frac{\omega_{C1}}{j\omega}\right)}\right)\left(\frac{1}{1+\left(\frac{\omega_{C2}}{j\omega}\right)}\right)$$

Series Capacitance and low frequency response



in order to find ω_L, we find magnitude of the gain at ω_L

$$|A(j\omega_L)| = \frac{Am}{\sqrt{2}}$$

solving yields

$$\omega_L^{\,2} = \frac{\omega_{C1}^{\,2} + \omega_{C2}^{\,2}}{2} + \frac{\sqrt{\omega_{C1}^{\,4} + 6\omega_{C1}^{\,2}\omega_{C2}^{\,2} + \omega_{C2}^{\,4}}}{2}$$

Series Capacitance and low frequency response

1) let $\omega_{C1} = 616$ rad/sec and $\omega_{C2} = 17.86$ rad/sec

here $\omega_{C1} >> \omega_{C2}$

$\omega_L = 616.5$ rad/sec

2) let $\omega_{C1} = 200$ rad/sec and $\omega_{C2} = 750$ rad/sec

here $\omega_{C2} >> \omega_{C1}$

$\omega_L = 798$ rad/sec

$$\omega_L^2 = \frac{\omega_{C1}^2 + \omega_{C2}^2}{2} + \frac{\sqrt{\omega_{C1}^4 + 6\omega_{C1}^2\omega_{C2}^2 + \omega_{C2}^4}}{2}$$

In both cases and in general

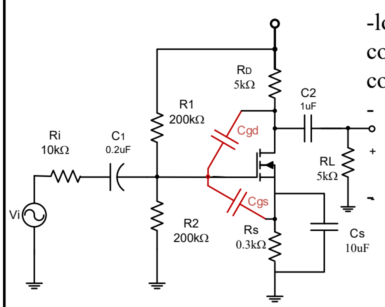
if $\omega_{C1} >> \omega_{C2}$

$\omega_{C1} < \omega_L < \omega_{C1} + \omega_{C2}$

Biggest $\omega_{C1} < \omega_L <$ sum of all ω_C's

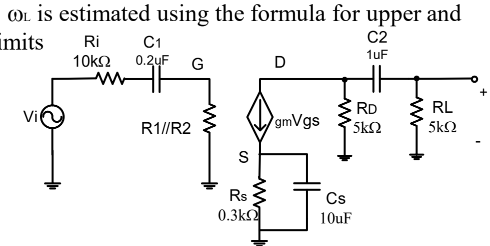
Low Frequency Response Example

- Calculate the low frequency corner frequencies due to C1,C2, Cs and estimate ω_L ?



-low frequency ac small signal equivalent circuit is constructed (here all high frequency caps Cgs, Cgd are considered as open circuits)

- We consider one capacitor each time , while all other low frequency caps are considered short circuit and its corresponding ω is found
- Finally, ω_L is estimated using the formula for upper and lower limits



Effect of each Capacitor at ω_L

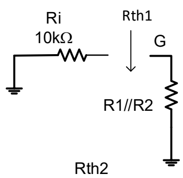
- We Calculate the low frequency corner frequencies due to each cap acting alone while all others are considered as short circuit

1) consider C1 (while C2 and Cs are shorted)

$$\omega_{C1} = \frac{1}{C1.Rth1} = 45.45 \text{ rad/sec;}$$

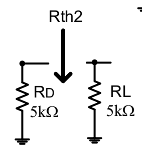
*Rth*1 is the thevenin impedance seen by C1 while all independant sources are set to zero

$$Rth1 = Ri + (R1//R2)$$



2) consider C2 (while C1 and Cs are shorted)

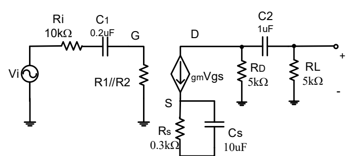
$$\omega_{C2} = \frac{1}{C2.Rth2} = 100 \text{ rad/sec;}$$



*Rth*1 is the thevenin impedance seen by C1

$$Rth2 = R_D + R_L$$

Effect of each Capacitor & ω_L



- We Calculate the low frequency corner frequencies due to cap acting alone while all others are considered as short circuit

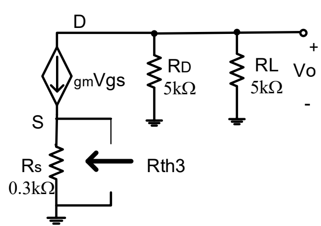
3) consider Cs (while C1 and C2 are shorted)

$$\omega_{C3} = \frac{1}{Cs.Rth3} = 1050 \text{ rad/sec;}$$

*Rth*3 is the thevenin impedance seen by Cs

remember $rds = \infty$

$$Rth3 = Rs // \frac{1}{gm}$$



4) estimation of the ω_L

$$1050 < \omega_L < 1195.5$$

Design of ω_L

- Previous method explained how to estimate value of ω_L in an analysis problem where all capacitor values are given, but what happens if it was desired to design an amplifier with certain ω_L and the task was to find capacitor values ?
- Design criteria to be used is:

$$\omega_{CE} = (0.7 - 0.8)\omega_L$$

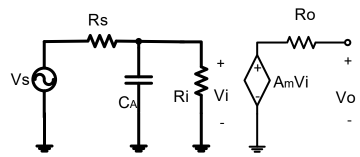
$$\omega_{C1} = \omega_{C2} = (0.1 - 0.15)\omega_L$$

C1,C2 are input and output coupling capacitors

C_E is bypass capacitor // to R_E emitter stabilizing resistor or Rs source resistor

make sure that $\omega_{CE} + \omega_{C1} + \omega_{C3} = \omega_L$

Shunt Capacitance and High frequency response



$$Vo = AmVi$$

$$Vi = \frac{Ri // \frac{1}{j\omega C_A}}{\left(Ri // \frac{1}{j\omega C_A}\right) + Rs} Vs \;\rightarrow\; \frac{Vo}{Vs} = A(j\omega)$$

$$A(j\omega) = Am \frac{Ri // \frac{1}{j\omega C_A}}{\left(Ri // \frac{1}{j\omega C_A}\right) + Rs}$$

$$= Am\left(\frac{Ri}{Ri + Rs}\right)\left(\frac{1}{1 + j\omega C_A(Rs // Ri)}\right)$$

$$\rightarrow |A(j\omega)| = Am \frac{Ri}{Ri + Rs} \cdot \frac{1}{\sqrt{1 + [\omega C_A(Rs // Ri)]^2}}$$

$$\rightarrow |A(j\omega)| = Av(mid)\frac{1}{\sqrt{1 + [\omega C_A(Rs // Ri)]^2}}$$

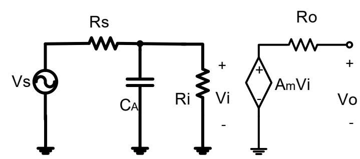
$$\rightarrow |A(j\omega)| = Av(mid)\frac{1}{\sqrt{1 + \left[\frac{\omega}{\omega_{CA}}\right]^2}}$$

$\rightarrow$ at $\omega = \omega_H = \omega_{CA}$

$$\therefore |A(j\omega_H)| = Av(mid)\frac{1}{\sqrt{2}}$$

$$\rightarrow \omega_H C_A(Rs // Ri) = 1$$

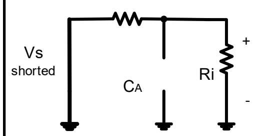
Shunt Capacitance and High frequency response



$$\rightarrow \ \omega_H C_A (Rs // Ri) = 1$$

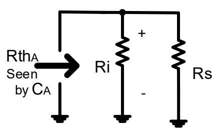
$$\therefore \ \omega_H = \frac{1}{C_A (Rs // Ri)}$$

where Rth_A is thevenin impedance seen by capacitor C_A



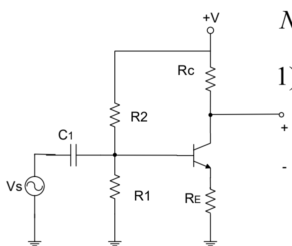
For Av(mid) = 1

for $\omega = \omega_{CA} \rightarrow$ 20log |A(jω)| = 20logAv(mid) - 20log0.707 = - 3dE



for $\omega = 10\omega_{CA} \rightarrow$ 20log |A(jω)| = - 20log10 = - 20dB ← low pass filter

Shunt Capacitance and High frequency response



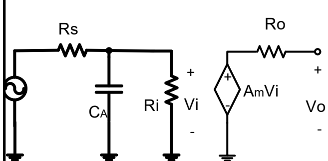
Note:

1) If there is only one cap, we find $\omega_{CA} = \frac{1}{R_{thA}.C1}$ and $\omega_H = \omega_{CA}$

2) If there is two caps C_A and C_B with ω_{CA} and ω_{CB}, then

$$A(j\omega) = Av(mid)\frac{1}{1+\left(\frac{j\omega}{\omega_{CA}}\right)}\frac{1}{1+\left(\frac{j\omega}{\omega_{CB}}\right)}$$

in order to find ω_H, we find magnitude of the gain at ω_H



$$|A(j\omega_H)| = \frac{Av(mid)}{\sqrt{2}} = \frac{Av(mid)}{\left|\left(1+\left(\frac{j\omega}{\omega_{CA}}\right)\right)\left(1+\left(\frac{j\omega}{\omega_{CB}}\right)\right)\right|}$$

Shunt Capacitance and High frequency response

By solving for the magnitude of the gain $A(j\omega)$ at $\omega = \omega_H$ yeilds for an approximation for the lower and upper limit to estimate ω_H for $\omega_{CA} >> \omega_{CB}$

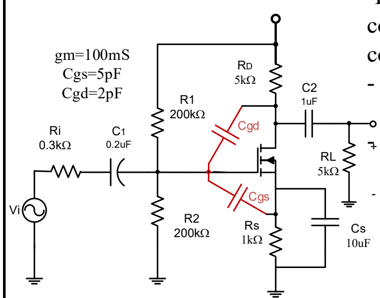
$$\frac{1}{\frac{1}{\omega_{CA}} + \frac{1}{\omega_{CB}}} < \omega_H < \omega_{CB}$$

$$\frac{\omega_{CA} . \omega_{CB}}{\omega_{CA} + \omega_{CB}} < \omega_H < \omega_{CB}$$

$$\text{lower limit} < \omega_H < \text{Smallest } \omega$$

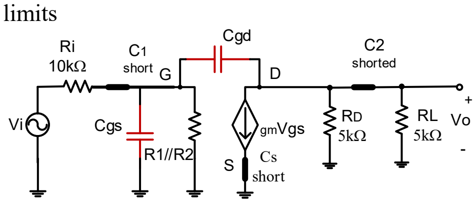
High Frequency Response Example

- Calculate the high frequency corner frequencies due to Cgs,Cgd and estimate ω_H ?



-High frequency ac small signal equivalent circuit is constructed (here all low frequency caps C1, C2 and C3 are considered as short circuits)

- We consider one capacitor each time , while all others are considered open circuit and its corresponding ω is found Finally, ω_H is estimated using the formula for upper and lower limits



Effect of each Capacitor & ω_H

- We Calculate the high frequency corner frequencies due to each high frequency cap acting alone while all others are considered as open circuit

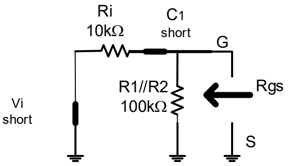
1) Consider Cgs (while Cgs is open , C1, C2 & Cs are shorted)

$$\omega_{Cgs} = \frac{1}{Cgs.Rgs}$$

Rgs is the thevenin impedance seen by Cgs

Rgs = R1//R2//Ri

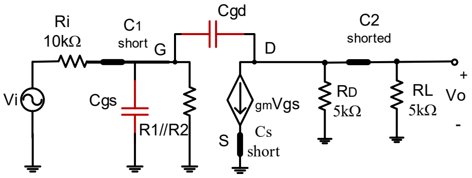
ω_{Cgs} = 668.45 Mrad/sec;



Effect of each Capacitor & ω_H

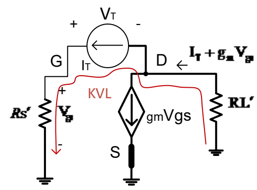
2) Consider Cgd (while Cgs is open , C1, C2 & Cs are shorted)

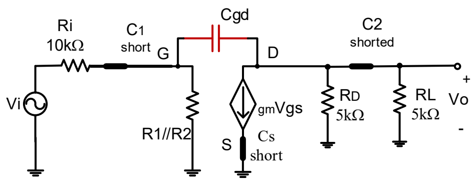
$$\omega_{Cgd} = \frac{1}{Cgd.Rgd}$$

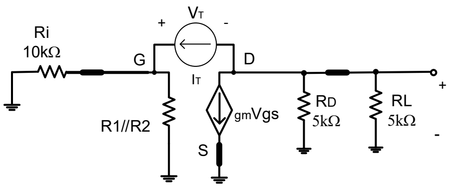


Effect of Capacitor Cgd

- Calculation of Rgd is done through test current /voltage method







KVL :

$$RL'(I_T + g_m V_{gs}) + I_T Rs' = V_T$$

but $V_{gs} = V_g - V_s = Rs' I_T$

substituting yeilds

$$RL'(I_T + g_m Rs' I_T) + I_T Rs' = V_T$$

$$Rgd = \frac{V_T}{I_T} = RL' + Rs' + g_m RL' Rs'$$

$$RL' = R_D // R_L \quad \text{and} \quad Rs' = Ri//R1//R2$$

Effect of each Capacitor & ω_H

Now

$$\omega_{Cgd} = \frac{1}{Cgd.Rgd} = 48.54 \text{ Mrad/sec;}$$

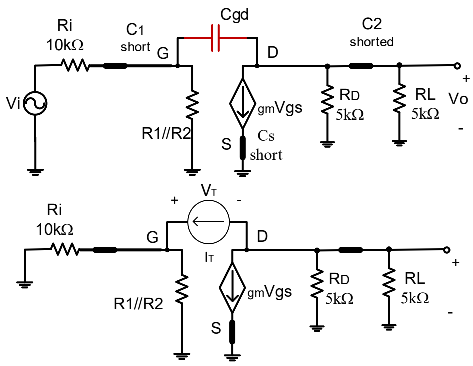
3) Estimation of the ω_H

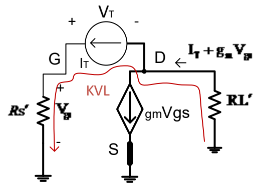
$$\frac{(\omega_{gd} \bullet \omega_{gs})}{(\omega_{gd} + \omega_{gs})} < \omega_H < 48.45$$

$$45.25 < \omega_H < 48.45$$

Effect of each Capacitor & ω_H

- Calculation of Rgd is done through
- test current /voltage method





$$KVL\ :RL'(I_T + g_m V_{gs}) + I_T Rs' = V_T$$

$$\text{but}\quad V_{gs} = V_g - V_s = Rs' I_T$$

substituti ng yeilds

$$RL'(I_T + g_m Rs' I_T) + I_T Rs' = V_T$$

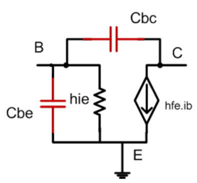
$$Rgd = \frac{V_T}{I_T} = RL' + Rs' + g_m RL' Rs'$$

$$RL' = R_D // R_L \quad \text{and} \quad Rs' = Ri//R1//R2$$

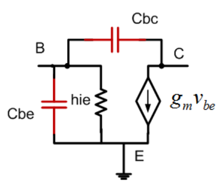
BJT High Frequency Response

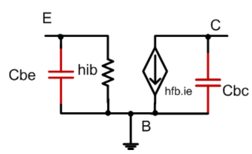
- Capacitors Cbe and Cbc
- CE and CC model

CB model



Covert to a form similar to FET





hfb= α=-1

$$h_{fe}.i_b = h_{fe}\frac{v_{be}}{h_{ie}} = g_m v_{be};$$

where

$$\frac{h_{fe}}{h_{ie}} = g_m$$

CE Example:

- Estimate the high corner frequency for the following BJT amplifier

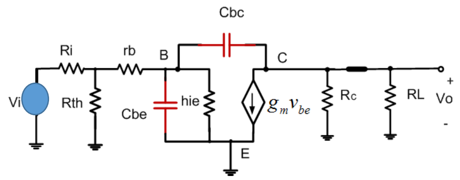
1) Effect of Cbe (Cbc is considered open)

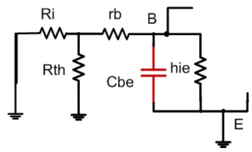
High Frequency Small Signal equivalent Circuit

$$\omega_{be} = \frac{1}{C_{be}.R_{be}};$$

where R_{be} is the thevenin impedance seen by C_{be}

$$R_{be} = ((Ri // Rth) + rb) // hie$$



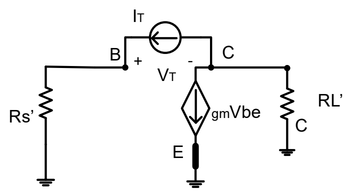


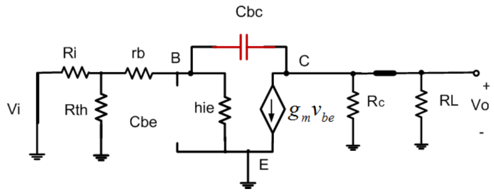
CE Example:

2) Effect of Cbc (Cbe is considered open)

$$\omega_{bc} = \frac{1}{C_{bc}.R_{bc}};$$

where R_{bc} is the thevenin impedance seen by C_{bc} and it is found by V_T/I_T method





$$RL'(I_T + g_m V_{be}) + I_T Rs' = V_T$$

but $V_{be} = V_b - V_e = Rs' I_T$

substituting yeilds

$$RL'(I_T + g_m Rs' I_T) + I_T Rs' = V_T$$

$$Rbc = \frac{V_T}{I_T} = RL' + Rs' + g_m RL' Rs'$$

$$Rs' = (Ri//Rth + rb) // hie$$

CE Example:

- Given the following values in previous example

$g_m = 33.5\ mS$

$hie = 8.77\ k\Omega$

$hfe = 294$

$Rs = 1\ k\Omega; R1//R2 = 16.67\ k\Omega$

$rb = 20\ \Omega; Cbc = 1.8\ pF; Cbe = 17.25\ pF$

$Rc = 5\ k\Omega \quad ; RL = 2\ k\Omega$

calculate :

$$\omega_{bc} = \frac{1}{C_{bc}.R_{bc}} = 66.7\ \text{Mrad/sec}$$

$$\omega_{be} = \frac{1}{C_{be}.R_{bc}} = 12.67\ \text{Mrad/sec}$$

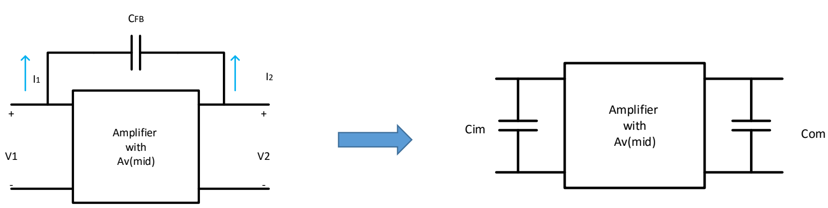
Estimate ω_H

$$\frac{(\omega_{be} \bullet \omega_{bc})}{(\omega_{be} + \omega_{bc})} < \omega_H < \omega_{be}$$

$$10.65 < \omega_H < 12.67$$

Miller Theorem (another method to solve previous example)

- Miller theorem is used to simplify the analysis of inverting amplifiers only at high frequencies
- The feedback capacitor Cbc or Cgd is decomposed into two capacitors , one at the input Cim and one at the output Com , whose values are found using the following formulas:



Input Miller Capacitance

$$C_{IM} = C_{FB}[1 - Av(mid)]$$

Output Miller Capacitance

$$C_{OM} = C_{FB}\left[1 - \frac{1}{Av(mid)}\right]$$

CE Example using Miller Theorem:

- Estimate the high corner frequency for the following BJT amplifier using miller theorem

- Calculate $A_V(\text{mid}) = \frac{Vy}{Vx}$

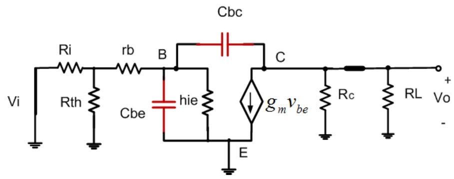
- Calculate $C_{IM} = C_{FB}[1 - Av(\text{mid})]$; $C_{FB} = C_{BC}$

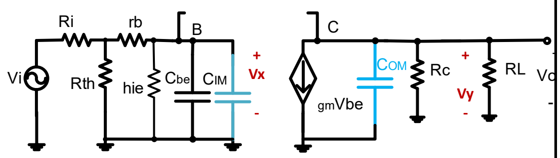
- Calculate $C_{OM} = C_{FB}\left[1 - \frac{1}{Av(\text{mid})}\right]$

- Calculate $\omega_{IM} = \frac{1}{(C_{IM} + C_{be})R_{be}}$

- Calculate $\omega_{OM} = \frac{1}{C_{OM}R_{ce}}$

- Estimate ω_L





CB Example :

- Estimate the high corner frequency ω_H for the following BJT amplifier

1) Effect of Cbe (Cbc is considered open)

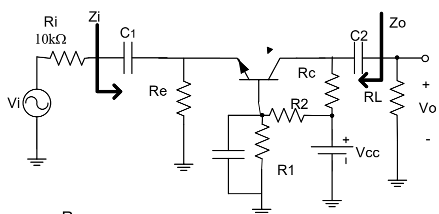
$\omega_{be} = \frac{1}{C_{be}.R_{be}}$; where R_{be} is the thevenin impedance seen by C_{be}

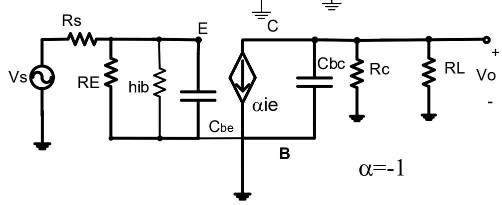
$R_{be} = ((Rs // RE)) // hib$

2) Effect of Cbc

$\omega_{bc} = \frac{1}{C_{bc}.R_{bc}}$;

where $R_{bc} = R_L // R_C$





α=-1

Homework : Amplifier Frequency Response

- Estimate the value of **low** and **high** frequency corner frequencies and calculate the mid-range voltage gain of the following amplifier

Important